Time-Optimized North-South Stationkeeping

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Introduction

THE long-period perturbations of the solar-lunar gravitational fields cause a steady precession of the orbital plane and exhibit a large number of feriodic terms having different frequencies appearing in the orbital elements. For a nearly equatorial satellite orbit this increase will average 0.86 deg annually. In order to effectively control the performance of a geosynchronous satellite, plane change maneuvers (north-south stationkeeping) must be performed to minimize peak latitude excursions. It is not atypical for this need to account for 95% of maneuver propulsion. Typically these maneuvers are performed at the ascending node of the orbit and as a consequence of the periodic regression of the lunar orbital nodes the next maneuver is likely to occur earlier than operationally desired. A control strategy is developed here that will produce the maximum time between maneuvers and the minimum annual velocity change.

Solar-Lunar Gravitational Perturbations

In order to eliminate the singularities present in the Keplerian angular elements when inclination and eccentricity are small, a modified set of elements may be used. The elements eccentricity (e), inclination (i), argument of perigee (ω), right ascension of the ascending node (Ω), and mean anomaly (M) are to be replaced by

$$h_1 = e\sin(\omega+\Omega) \qquad h_2 = \sin i \sin\Omega$$

$$k_1 = e\cos(\omega+\Omega) \qquad k_2 = \sin i\cos\Omega \qquad \Lambda = \omega+\Omega+M \tag{1}$$

Analysis of the disturbing functions can yield the rate of change for each nonsingular element (E_i) as a series of the form

$$\mathrm{d}E_i/\mathrm{d}t = A_i + \sum_{j=1}^{N} B_{ij}\cos(\nu_j t) + C_{ij}\sin(\nu_j t) \tag{2}$$

or alternatively by numerical integration of the equations of motion.[1-3] The N frequencies, ν_j, are linear combinations of the satellite, solar, or lunar orbital frequencies. The constants A_i, B_{ij}, and C_{ij} are best determined by a least-squares fit to the results of the rates of change of the nonsingular elements computed from the direct numerical integration of the equations of motion.

If inclination is suitably small ($\leq 10^{-1}$ rad) then $i \approx \sin i$, and the nonsingular elements h_2 and k_2 are written as

$$h_2 = i\sin\Omega \qquad k_2 = i\cos\Omega \tag{3}$$

and can be represented as a complex quantity

$$\boldsymbol{i} = ie^{j\Omega} \tag{4}$$

Physically this represents the orientation of the orbit normal, with the real part directed along a line to the first point of Aries, and the imaginary part perpendicular to the line to the first point of Aries in the equatorial plane. Additionally, this

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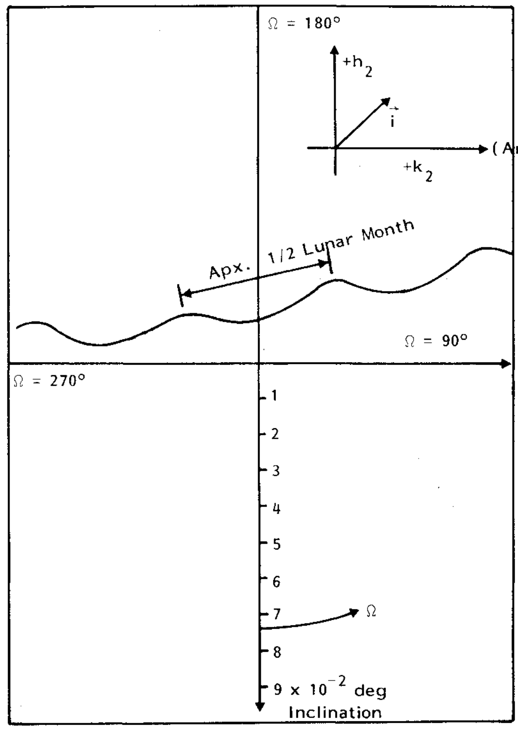

Fig. 1 Typical inclination trajectory.

allows a polar plot to be made of the vector, such as that in Fig. 1. The direction of the precession is generally toward Aries, but there can be a deviation of ± 9 deg during the cycle. This deviation exhibits a period of one-half a solar year.

The principal motion in inclination varies from 0.0134 to 0.0169 rad/yr, with the 18.6 yr lunar period. The principal lunar term in the series from Eq. (2) is

$$\mathrm{d}k_2/\mathrm{d}t = -6.196\times10^{-6}\cos\lambda_m \text{ rad/day}$$

$$\mathrm{d}h_2/\mathrm{d}t = 4.1015\times10^{-5} + 4.7997\times10^{-6}\cos\lambda_m \text{ rad/day}$$

where λ_m is the longitude of the lunar ascending node. These terms produce a minimum inclination variation in 1978 (0.769 deg) and a maximum variation in 1987 (0.968 deg). As a result of these variations, periodic maneuvers must be performed in order to keep inclination below some value $i_{\max}$.

Control Strategy

Typically, maneuvers have been performed whenever the magnitude of the inclination is at or near the constraining limit $i_{\max}$. This has involved maneuvers whose goal vectors, $\boldsymbol{i}_g$, are

$$\boldsymbol{i}_g = -\boldsymbol{i}$$

at the appropriate point in the orbit. Over a period of time this procedure produces a series of trajectories much like that in Fig. 2. The net effect is to cause maneuvers to occur at closer and closer intervals. This clearly is a poor way to control inclination. As an alternative, consider the following.

Suppose that a suitable inclination limit, $i_{\max}$, and a minimum measurable inclination, $i_{\min}$, are known. This minimum measurable inclination could also be thought of as a limiting measurement noise, and may be of the order 10^{-5} rad. Consequently, a maneuver control strategy will be developed that permits the inclination trajectory to have a magnitude less than or equal to $i_{\min}$ somewhere near the middle of its path. That is, on a polar plot the trajectory will pass through a circle of radius $i_{\min}$ centered at the origin.

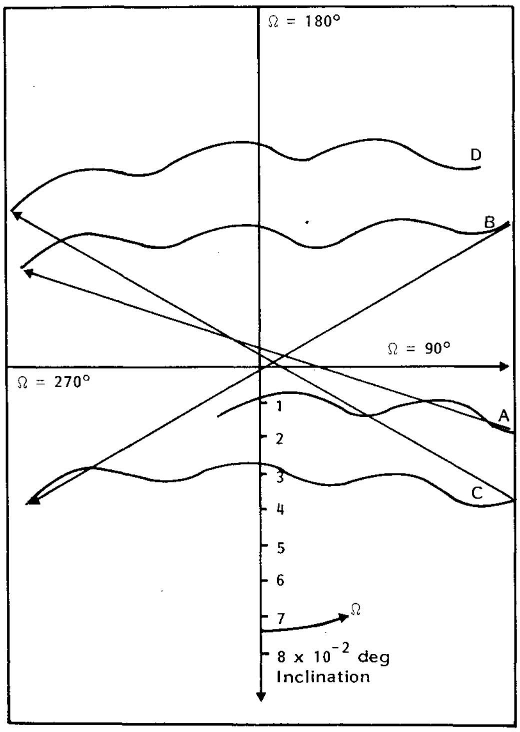

Fig. 2 Typical control history.

If the initial vector is i_0 at time T_0, then the estimated goal vector becomes

$$i_g = -i_{\max}\hat{i}_0 = i_{\max} e^{j(\Omega_0+\pi)} \tag{5}$$

This vector is propagated from time T_0 to time T_N via the series expansion from Eq. (2) to form the vector $i'(t)$. The value of time T_N is determined by

$$T_N = T_0 + (P_m/2)\,(i_{\max}+i_{\min})/\,|i_g - i'\,(T_0+P_m/2)\,| \tag{6}$$

where the denominator represents the magnitude of the inclination motion during one-half of a lunar month, P_m. As the vector $i'(t)$ is being propagated from $t=T_0$ to $t=T_N$, a comparison test is made

$$|i'(t)| \le i_{\min} \tag{7}$$

If the results of this test are positive, then the goal vector has indeed been found, otherwise the estimated goal vector must be modified as follows.

Suppose that in the course of this comparison note has been made of the inclination closest to the origin, i_c. Thus an imaginary triangle can be formed whose sides are i_g, $i_g - i_c$, and i_c. Furthermore, another triangle can be formed whose sides are i_g, $i_g - i_{\min}\hat{i}_c$, and $i_{\min}\hat{i}_c$. Figure 3 is representative of this discussion. Thus vector algebra can be used to determine the angle, $\delta\Omega$, which must be added to Ω_g (initially set to $\Omega_0+\pi$) to allow the trajectory to pass through the circle of radius $i_{\min}$. That is

$$\delta\Omega = \tan^{-1}\{\,(\,|i_g - i_c) \times (i_g - i_{\min}\hat{i}_c)\,|\,/\,(i_g - i_c)\cdot(i_g - i_{\min}\hat{i}_c)\,\} \tag{8}$$

Consequently a new inclination goal vector is determined

$$i_g = i_{\max} e^{j\Omega_{g\mathrm{NEW}}} = i_{\max} e^{j(\Omega_0+\pi+\delta\Omega)} \tag{9}$$

for the first iteration. For subsequent iterations let $\Omega_{g\mathrm{NEW}} = \Omega_{g\mathrm{OLD}} + \delta\Omega$ and the earlier procedures are repeated until the condition in Eq. (7) is fulfilled.

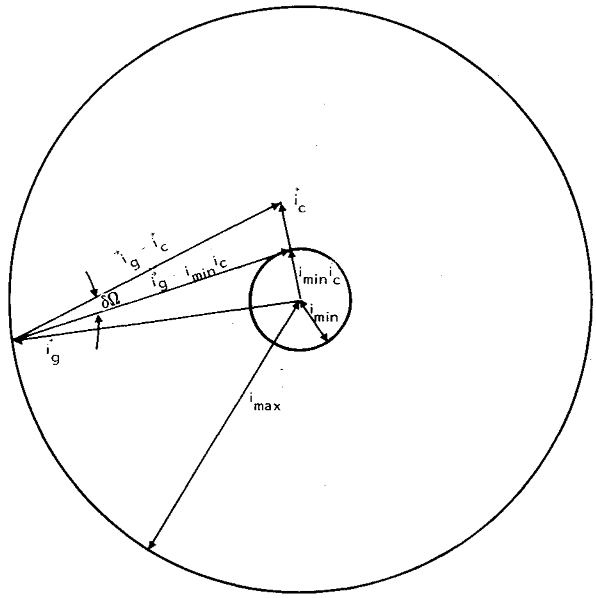

Fig. 3 Control strategy triangles.

Having determined the goal inclination vector, the velocity impulse and the true anomaly of its application are easily determined. The change in inclination, Δi, is

$$\Delta i = i_0 - i_g \tag{10}$$

and the velocity impulse becomes

$$\Delta V = 2V_\theta \sin(\Delta i/2) \tag{11}$$

where V_θ is the orbital velocity at the true anomaly for the thrust application. The true anomaly and the resultant time of day at which to perform the maneuver can be found by the test

$$\hat{r}_0 \cdot \hat{h}_g = \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} \cdot \begin{bmatrix} h_2 \\ -k_2 \\ \cos i_g \end{bmatrix} = 0 \tag{12}$$

where $\hat{r}_0$ is the unit vector radius of the initial orbit, and $\hat{h}_g$ is the orbit normal for the goal orbit.[4] Note that this search for the time will require propagation of the initial orbital parameters. This can be accomplished analytically or by numerical integration of the complete equations of motion.

Optimization

The technique outlined in the control strategy section attempts to find an inclination trajectory with the greatest arc length through the constraint circle of radius $i_{\max}$. This is to be contrasted to other procedures which do not attempt to find this path. In view of the fact that the perturbations are the same in either case, the maximum arc length stratagem will also be a maximum elapsed time stratagem (i.e., the time between maneuvers has been optimized).

This technique also has another benefit. The minimum inclination to be removed is primarily given by the principal lunar terms, so that the minimum annual velocity change is given by

$$\Delta V_{\min} = 2V\sin(\Delta i_{\mathrm{yr}}/2)$$

where

$$\Delta i_{\mathrm{yr}} = \left|\left(\int_0^{365} \frac{\mathrm{d}k_2}{\mathrm{d}t}\,\mathrm{d}t,\ \int_0^{365} \frac{\mathrm{d}h_2}{\mathrm{d}t}\,\mathrm{d}t\right)\right| \tag{13}$$

The definitions for the time derivatives of k_2 and h_2 can be found in an earlier section. The functional behavior of Δi_{yr} is exactly that of the control strategy. Consequently, the technique also has determined the minimum annual velocity change required. As velocity change (delta-V) is directly related to propellant consumption, the minimum amount of consumable has also been found.

Conclusion

A procedure or control strategy has been formulated that produces a trajectory history permitting the maximum time between maneuvers given a constraining limit. Consequently, this also leads to the optimum (i.e., minimum) annual delta-V requirements.

There is considerable payoff from this procedure. The technique developed follows the slowly varying precession toward Aries, whereas other techniques do not. Use of these techniques has a price in the form of goal vectors in error with respect to the optimum. This error will average about 4.5 deg corresponding to increased velocity needs as much as 7% greater. For a fixed propellant load, the technique described in the control strategy section will extend mission life by this same amount (i.e., up to 6 months additional life for a 7-yr vehicle), in contrast to other strategies.

References

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Roll Resonance Probability for Ballistic Missiles with Random Configurational Asymmetry

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Introduction

ROLL resonance during ballistic flight normally is a routine, uneventful state in which the missile spin rate briefly becomes resonant with the pitch rate. However, under certain conditions these two time-varying frequencies will converge and tend to remain stable with respect to each other for an extended period. When resonance persists, the aerodynamic trim angle is amplified. In response, the angle of attack rapidly increases, particularly in the case of small bodies, and as a result of the increased drag, trajectory dispersion suffers. Sometimes the lateral force developed on the vehicle becomes excessive and causes catastrophic failure. Consequently, the roll resonance event is a major design consideration of uncontrolled flight.[1]

Seemingly symmetrical missiles, including finless bodies of revolution such as conventional artillery shells and re-entry vehicles, produce small but not always negligible roll torques. The configurational irregularities, which can be brought about either by in-flight distortions caused by aerodynamic heating or loading or by manufacturing tolerances and imperfections, induce a body-fixed side force that combines with an offset or misaligned mass axis to create a moment about the vehicle roll axis. If the moment acts in a favorable direction, the resulting angular acceleration serves to further separate the roll and pitch frequencies, and an ordinary transient resonance situation takes place. In the other direction, a rolling moment can sustain the resonance mode when the torque exceeds a critical amount (L_{crt}).

In a group of supposedly identical missiles, the aerodynamic and mass asymmetries will vary from body to body in some random fashion. The sizes of the rolling moments induced by the slight configurational disorders depend on the magnitudes and relative orientations of the two factors that comprise each of the moments: the lateral force due to a nonzero aerodynamic trim angle, and a lever arm arising from nonuniform mass distribution. With a fixed degree of maximum possible asymmetry and specified flight conditions, the moments will range over the interval $\pm L$, where L denotes the finite limiting value. If the possible asymmetry becomes excessive, i.e., capable of prolonging a coupling of the roll and pitch frequencies ($|L| > |L_{crt}|$), some, but not all, of the bodies will possess a moment of sufficient size and direction to maintain resonance. Hence, a problematical or uncertain condition of resonance prevails.

To quantify a missile's susceptibility to the resonance state, an integral equation was written that describes the likelihood of the occurrence of continuous roll resonance, assuming that the asymmetry moment evolves randomly without bias. The derivation of the probability equation is presented along with numerical solutions in the following discussion. No other record of resonance probability theory is known.

Discussion

The angular acceleration in the roll direction will not be sufficient to sustain a permanent state of resonance when the torque resulting from configurational abnormalities is dominant and remains less than a critical amount.† Therefore, the probability of continuous resonance is zero for missiles with negligible asymmetry; i.e., $P=0$ if $|L| < |L_{crt}|$.

When the asymmetry moment becomes excessive, resonance may occur. Assuming that uniform flight conditions are maintained and that the angle of attack has stabilized at the trim condition, the boundary that defines the circumferential region in which a lateral center-of-mass position vector $\boldsymbol{\delta}$ will be situated during the inception of resonance is readily determined from the problem geometry. If the contribution to the trim angle that arises from mass asymmetry effects is ignored, the angle $\zeta=\cos^{-1}|\delta_{crt}|/|\delta|$ demarcates the resonance zone in the case of out-of-plane asymmetries (see Fig. 1).

The determination of the probability of resonance now becomes a matter of asymmetry apportionment. An elementary case involves a group of nearly identical missiles

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†Numerical studies can be conducted with arbitrary flight conditions using a multidegree-of-freedom trajectory computer code to bracket the critical moment by a trial-and-error process. Approximate, closed-form expressions of roll resonance criteria, from which the critical value may be estimated, are developed for out-of-plane asymmetries by Vaughn[2] and for in-plane asymmetries by Barbera.[3] *Out-of-plane* asymmetries refer to an arrangement in which the planes of the trim angle and the cm position vector are mutually perpendicular. These two planes contain the body longitudinal axis and are coincident with in-plane asymmetries.